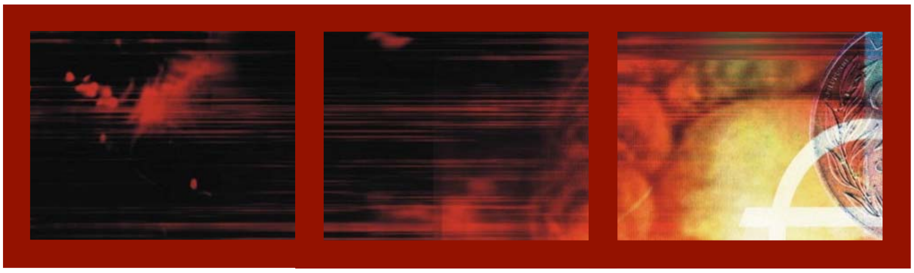

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The Omega Function

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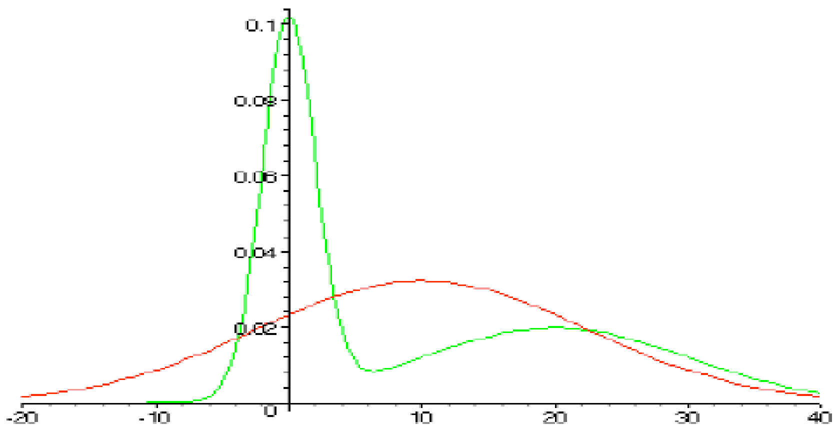
This article presents a brief introduction to the Omega function. The Omega function is a performance measurement function recently introduced in 2002 by William F. Shadwick and Con Keating. Unlike more traditional performance measurements such as the Sharpe ratio which necessitate the assumption of a normal distribution of returns, the Omega function can be applied to asymmetrical return distributions.

Exhibit 1



(Source: Keating and Shadwick)

The two return distributions in Exhibit 1 above have the same mean and variance. A performance comparison using Sharpe Ratio analysis which focuses only on mean and variance and assumes all distributions are normally distributed would conclude that these two distributions are equivalent. Simple inspection of their graphical plots clearly indicates that their risk/return characteristics are not equivalent. The Omega function analysis does not assume a normal distribution and allows for a more accurate analysis of risk/return characteristics of non-normal asymmetrical return distributions such as the distributions in Exhibit 1.

The Sharpe ratio is one of the most widely used investment performance measures. Recall that the Sharpe ratio[1] is calculated by subtracting the risk-free rate from the rate of return of a portfolio and dividing this by the standard deviation of the portfolio returns. Unfortunately because the Sharpe ratio requires the assumption that return distributions are normal distributions, the Sharpe ratio is not always the best measure of investment performance in the case of portfolio returns that are not normally distributed such as in the case of hedge fund and fund of funds portfolio return distributions. In the case of non-normal and asymmetrical return distributions, the Omega function is a more appropriate performance measure.

[1] Sharpe ratio = $\frac{(E(R) - RFR)}{SD}$

The Omega function is an investment performance measure that allows for the assessment of asymmetrical return distributions such as one finds in hedge funds and funds of hedge funds. As opposed to the Sharpe ratio which assumes a normal distribution of returns, the Omega function requires no such assumption and allows for the assessment of asymmetrical return distributions. The Omega function is defined as the probability weighted ratio of gains to losses relative to a threshold return level as determined by the investor such as the risk-free rate. Simply put, the Omega is the ratio of probability weighted gains to probability weighted losses. The Omega function calculation partitions returns into losses and gains above and below a return threshold and then takes the probability weighted ratio of returns above the return threshold (i.e. probability weighted gains) divided by the returns below the threshold (i.e. probability weighted losses). Whereas the Sharpe ratio defines risk in terms of the standard deviation of an assumed normal distribution of individual returns without regard to upside or downside volatility of returns, Omega defines risk with respect to the downside and upside volatility of returns comprising the entire return distribution of a portfolio and allows for asymmetrical return distributions. Unlike the Sharpe ratio, the Omega function does not require the assumption of a normal distribution or assumptions about investor risk preferences through the use of utility functions. Instead the Omega function is premised on the rule that more money is preferable to less money such that an asset or portfolio with a higher Omega value is preferable to one with a lower value.

Omega can be used to unequivocally rank and evaluate portfolio performance. The Omega function allows one to discern performance differences between multiple portfolios with asymmetric return distributions in contrast to Sharpe ratio analysis which often incorrectly does not differentiate between very different portfolio return distributions because of the Sharpe ratio's overly simplistic focus on only mean and variance and the assumption of a normal distribution of returns. Comparing the Omega functions for two or more portfolios over a range of returns effectively ranks their performance and risk profiles. The larger the Omega value, the higher the quality of the portfolio returns distribution. The higher Omega value return distribution will dominate the other distribution with lower Omega values.

Unlike Sharpe ratio performance analysis focused solely on mean and variance, Omega performance analysis better captures the risk and reward features in a performance return distribution. Omega focuses on the ranking of performance distributions with respect to risk and return. Risk and return is evaluated using Omega in order to reward positive fat tail performance above the mean; penalize negative fat tail performance below the mean; reward higher means and to adjust Sharpe ratio rankings based upon normal distribution assumptions. By plotting the Omegas of different skewed and asymmetric return distributions, it is easy to discern that different investment portfolios are not equivalent (as it would appear if one were to merely calculate Sharpe ratios) and that portfolios that are more skewed to the right at any loss thresholds with higher Omega values are preferable to other portfolios with lower Omega values. The Omega function determines the relative quality of a bet on a return above a given loss threshold for any portfolio return distribution.

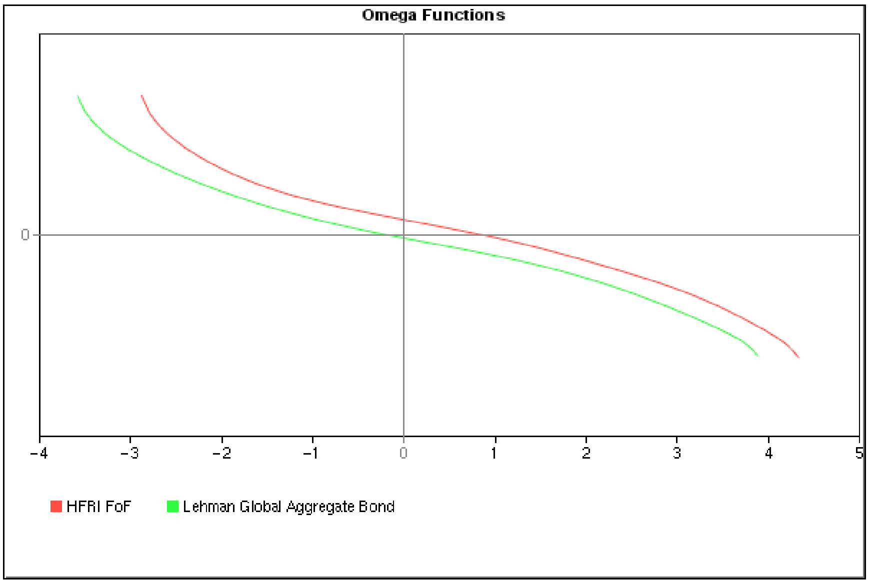
The two Omega function graphs in Exhibit 2 below illustrate how one may use the Omega function to compare and evaluate the performance distributions of two or more portfolios. Note that the Y Axis is the natural log (ln) of the probability adjusted ratio of gains over losses while the X Axis is the range of returns of the distribution of returns. Exhibit 2 graphs the Omega functions of the HFRI Fund of Funds ("FOF") Index in red and the Lehman Global Aggregate Bond Index in green. Simplistically, the upper left hand region represents the downside returns while the lower right hand region represents the upside returns. Where Y = 0, X represents the mean return of the distribution. Thus in our example, the mean return of the HFRI FOF Index is a little under 1% for

[2] Omega function =

$$\Omega(r) = \frac{\int_r^b (1 - F(x))dx}{\int_a^r F(x)dx}$$

this distribution while the mean return of the Lehman Bond Index is a little under 0%. The steepness of the curve represents the standard deviation of returns with a steeper curve representing lower standard deviation while a flatter curve represents a higher standard deviation. Looking at Exhibit 2, with respect to the negative returns in the upper left hand region, the HFRI FOF Index Omega function is steeper than that of the Lehman Bond index Omega function thus reflecting that the HFRI FOF index has a lower standard deviation of negative returns versus the Lehman Bond index. Similarly looking at the lower right hand region of Exhibit 2 for the positive returns of the distributions, one sees that the HFRI FOF Index Omega function is flatter than the Lehman Bond index which indicates that the standard deviation of positive upside returns is greater for the HFRI FOF Index than the Lehman Bond index. One may also compare the ranges of the negative returns and the ranges of the positive returns of the performance distributions by observing the Omega function coupled with their standard deviations (i.e. flatness implying higher standard deviations and steepness implying lower standard deviations) to assess the efficiency of one portfolio versus another. Referring again to Exhibit 2, one sees that the range of negative returns in the upper left hand region of the graph for the HFRI FOF Index in red is smaller than that of the Lehman Bond index in green while the HFRI FOF Index has a steeper graph in the negative return region than the Lehman Bond Index indicating that the HFRI FOF Index has a lower standard deviation of negative returns than that of the Lehman Bond Index. Looking at the positive return region in the lower right hand section of Exhibit 2, one can see that the range of positive returns for the HFRI FOF Index is greater than that of the Lehman Bond Index and that the Omega function is flatter for the HFRI FOF Index indicating a higher standard deviation of positive returns for the HFRI FOF Index versus the Lehman Bond Index. By assessing the downside return ranges with their respective standard deviations relative to the upside return ranges with their respective standard deviations as indicated by their respective Omega functions, one can quickly assess one portfolio versus another and would choose the portfolio that had a greater range and standard deviation of upside positive returns versus a lower range and standard deviation of downside negative returns. Observing the Omega functions, one sees that the HFRI FOF Index dominates the Lehman Bond Index in both the upside positive returns and volatility of returns (more upside positive returns) and the downside negative returns (less downside negative returns) thus indicating that the HFRI FOF Index portfolio is preferable to and so dominates the Lehman Bond Index portfolio with respect to risk and return.

Exhibit 2



References: *The following were used as references for this article*

1. *Con Keating and William F. Shadwick (2002), An Introduction to Omega.*
2. *Ana Cascon and William F. Shadwick (August 6, 2005), New Statistical Tools From Omega Functions.*
3. *William F. Shadwick (2004). Duke University, The Fuqua School of Business (PMRA), PowerPoint Presentation: Omega Metrics, The 21st Century Standard for Performance and Risk Management.*